NAG Toolbox for MATLAB

f08fn

1 Purpose

f08fn computes all the eigenvalues and, optionally, all the eigenvectors of a complex n by n Hermitian matrix A.

2 Syntax

```
[a, w, info] = f08fn(jobz, uplo, a, 'n', n)
```

3 Description

The Hermitian matrix A is first reduced to real tridiagonal form, using unitary similarity transformations, and then the QR algorithm is applied to the tridiagonal matrix to compute the eigenvalues and (optionally) the eigenvectors.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz – string**

If **jobz** = 'N', compute eigenvalues only.

If **jobz** = 'V', compute eigenvalues and eigenvectors.

Constraint: **jobz** = 'N' or 'V'.

2: **uplo – string**

If **uplo** = 'U', the upper triangular part of A is stored.

If **uplo** = 'L', the lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **a**(**lda**,$*$) **– complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n Hermitian matrix A.

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork, rwork

5.4 Output Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobz** = 'V', then if **info** $= 0$, **a** contains the orthonormal eigenvectors of the matrix A.

If **jobz** = 'N', then on exit the lower triangle (if **uplo** = 'L') or the upper triangle (if **uplo** = 'U') of **a**, including the diagonal, is destroyed.

2: **w($*$) – double array**

Note: the dimension of the array **w** must be at least $\max(1, \mathbf{n})$.

If **info** $= 0$, the eigenvalues in ascending order.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **uplo**, 3: **n**, 4: **a**, 5: **lda**, 6: **w**, 7: **work**, 8: **lwork**, 9: **rwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the algorithm failed to converge; i elements of an intermediate tridiagonal form did not converge to zero.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

Each eigenvector is normalized so that the element of largest absolute value is real and positive.

The total number of floating-point operations is proportional to n^3.

The real analogue of this function is f08fa.

9 Example

```
jobz = 'Vectors';
uplo = 'Upper';
a = [complex(1, +0), complex(2, -1), complex(3, -1), complex(4, -1);
     complex(0, +0), complex(2, +0), complex(3, -2), complex(4, -2);
     complex(0, +0), complex(0, +0), complex(3, +0), complex(4, -3);
     complex(0, +0), complex(0, +0), complex(0, +0), complex(4, +0)];
[aOut, w, info] = f08fn(jobz, uplo, a)
```

```
aOut =
    0.3839 + 0.2941i   -0.3975 + 0.5105i    0.3746 + 0.2414i    0.3309 -
0.1986i
    0.4512 - 0.1102i    0.3953 - 0.3238i   -0.2895 + 0.4917i    0.3728 -
0.2419i
   -0.0263 - 0.4857i   -0.4309 + 0.0383i   -0.3768 - 0.3994i    0.4870 -
0.1938i
  -0.5602            0.3648            0.4175            0.6155
w =
   -4.2443
   -0.6886
    1.1412
   13.7916
info =
           0
```